Coherent Particle Scatter in Smoke and Cumulus Clouds

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ABSTRACT

Recently, dual wavelength radar measurements of smoke and developing cumulus clouds have appeared in the literature which show a puzzling correlation between the measured radar returns. This correlation cannot easily be explained in terms of the conventional scattering mechanisms: coherent (Bragg) scattering from spatial fluctuations in clear-air refractive index and incoherent (Rayleigh) scattering from particles. In this paper, a possible explanation of the measurements is presented in terms of coherent scattering from the particles. It will also be argued that in clouds, Bragg scatter from fluctuations in water vapor content can be much smaller than from fluctuations in liquid water content, contrary to common belief.

INTRODUCTION

Two mechanisms important for the interpretation of radar observations of the atmosphere, are: coherent scattering from refractive index fluctuations of the clear-air and incoherent scattering from particles. These scattering mechanisms have very different wavelength dependencies and their contributions to the radar return can be separated by using two radars with different frequencies. Recent dual-frequency observations of smoke from an industrial fire [1] and developing cumulus clouds [2] show a puzzling correlation between the reflectivities. For the smoke, a scatterplot of X-band reflectivities versus UHF reflectivities showed that the points cluster around a line with a slope of roughly 2. The S-band and X-band reflectivities in developing cumulus clouds often cluster around a line with slope 1 and have a difference of about 10 dB. In this paper, an explanation will be proposed in terms of coherent scattering from particles. The paper is organized as follows. Firstly, some theory on coherent and incoherent scattering is reviewed. The puzzling measurements will be discussed next. The main emphasis will be on the smoke measurements. The cloud measurements of Knight and Miller are analyzed in more detail elsewhere [3].

Coherent scattering

Fluctuations in refractive index can be caused by turbulent mixing of air with different characteristics. For homogeneous, isotropic turbulence the radar reflectivity η due to backscattering from variations in refractive index is given by [4]:

$$\eta(k) = \frac{\pi^2}{2} k^4 \varphi_n(k) \qquad (1)$$

where k is the wavenumber which equals $4\pi/\lambda$ for back-scattering, λ being the radar wavelength. The three-dimensional spectrum of refractive index variations $\varphi_n(k)$ is given by:

$$\varphi_n(k) = 0.033 C_n^2 k^{-11/3} \qquad (2)$$

where C_n^2 is the *structure constant*. Substitution of (2) into (1) leads to the well-known formula:

$$\eta(\lambda) = 0.38 C_n^2 \lambda^{-1/3} \qquad (3)$$

The range of scales for which (2) is valid is called the *inertial subrange*. The largest scale in the inertial subrange is called the *outer scale* L_0 [5]. In the inertial subrange the fluctuations are passed on to smaller scales by the breaking-up of eddies, with a constant flux of variance. At the smallest scales the fluctuations are removed by molecular diffusion. The structure constant C_n^2 depends on the outer scale of the inertial subrange and on the variance of the refractive index. If it assumed that there are no fluctuations on scales larger than the outer scale L_0, then the following relation is obtained by integrating (2) over all scales in three dimensions:

$$C_n^2 = 8.7 L_0^{-2/3} \operatorname{var} n \qquad (4)$$

where var n is the spatial variance of the refractive index. The refractive index of air depends on pressure, temperature and water vapor mixing ratio, the latter being the most important contribution to the backscatter in the lower atmosphere [6]. The presence of small particles will change the refractive index of air as well. If the mutual distance between the particles is much smaller than the wavelength, the variance of the refractive index due to the presence of particles with diameter D and concentration N is given by [7]:

$$\operatorname{var} n = \frac{\pi^2}{16} |K|^2 \operatorname{var}(ND^3) \qquad (5)$$

where $|K|^2$ is a factor depending on the refractive index of the material of the particles. For water, $|K|^2$ equals about 0.93 for the wavelengths commonly used. If the number of particles is conserved and the particles follow the turbulent flow without influencing it, then (3) is also valid.

Incoherent scattering

The radar reflectivity for independently moving particles which are much smaller than the wavelength, is given by :

$$\eta(\lambda) = \frac{\pi^5 |K|^2}{\lambda^4} N D^6 \qquad (6)$$

For simplicity of notation, it is assumed in (5) and (6) that all the particles have the same diameter.

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Radar reflectivity factor

To compare measurements of rain and clouds performed with radars with different wavelengths and resolution volumes, the *radar reflectivity factor Z* is used:

$$Z = \frac{\lambda^4}{\pi^5 |K|^2}\eta \qquad (7)$$

For $|K|^2$ the value for water of 0.93 is taken. In the case of incoherent scatter from the particles, (6) must be substituted for η and the corresponding radar reflectivity factor will be denoted by Z_I . When coherent scattering from particles is considered, (3)-(5) must be used and the coherent radar reflectivity factor Z_C results. Z_I is independent of wavelength in the Rayleigh regime. Z_C is strongly dependent on wavelength. The radar return from a volume containing particles may have both an incoherent and a coherent component. This is because the theory of coherent scatter is derived for a *continuum*, whereas particles are discrete points. The Poisson statistics associated with the random positions of the individual particles cause an incoherent return.

Suppose air containing particles is mixing with air without particles. Both the mean concentration and the amplitude of fluctuations in the mixing region will depend linearly on the initial concentration. Therefore, one can assume that the standard deviation in the mass concentration of particles is proportional to the mean mass concentration, with a constant of proportionality which will be called β. The incoherent radar return will depend linearly on the initial concentration. The coherent return depends on the *variance* of the fluctuations and will therefore depend on the *square* of the concentration. Both the incoherent and coherent returns will depend on the diameter to the power of six. Combining (6) and (7) gives the incoherent radar reflectivity factor and combining (3)-(5) and (7) the coherent radar reflectivity factor. Combining incoherent and coherent terms, gives:

$$Z(\lambda) = ND^6 + 2.44\times10^{-2}\, L_0^{-2/3}\, \beta^2 N^2 D^6 \lambda^{11/3} \qquad (8)$$

In addition to the coherent scattering from the particles, there can also be a coherent component from the fluctuations in clear-air refractive index.

CORRELATED RADAR RETURNS

Smoke from an industrial fire

Rogers and Brown [1] have made measurements at wavelengths of 33 cm and 3.2 cm of the black smoke from a large industrial fire. The fire was 10 km away from the radars, and reached them after about 20 minutes. With these wavelengths, pure coherent scatter would give a 37 dBZ difference in the reflectivity factors and pure incoherent would give 0 dBZ. A mixture of incoherent scatter and coherent scatter would give a difference in between these values. A scatterplot of the reflectivity factors is shown in Fig. 1. It shows that almost all points lie between the lines of 0 dBZ difference and 37 dBZ difference. A puzzling thing is that there is a correlation between the reflectivity factors of both radars. The points lie roughly on a line of slope 2. Rogers and Brown investigate two possible explanations for the difference in the observed reflectivity factors: 1) the presence of cm-sized particles and 2) a strongly perturbed structure of atmospheric refractivity.

Large particles: There are indications that the smoke plume contained a small number of large particles. These indications are: a grainy pattern of high-resolution X-band reflectivity, a predominance of downward velocities in the order of 1 m/s and the fact that a Vaisala CT-12K ceilometer did not detect the plume. If the particles are so large that the Rayleigh approximation is not valid at X-band, then the UHF-band reflectivity will exceed the X-band reflectivity. Rogers and Brown show that a very small number of cm-sized particles can explain the magnitude of the differences that occur, but in order to explain the correlation in the measurements, the concentration of the particles must increase with their size, which seems unlikely.

It is puzzling that the ceilometer did not detect the plume, because even if the plume contained a small number of large particles, it would probably still contain small particles in much higher concentrations.

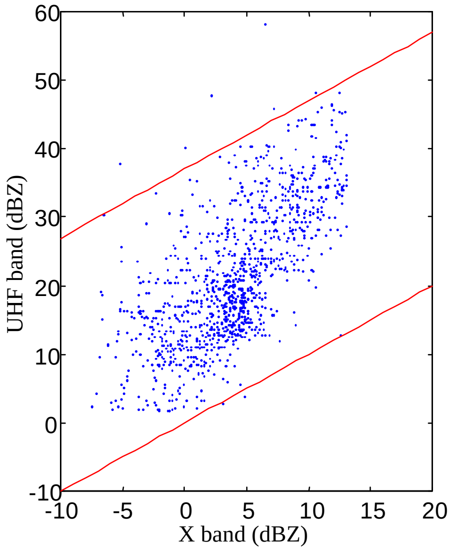
Strong clear-air scatter: The heat generated by the fire and gases that are produced could increase the value of C_n^2 to values considerably higher than ordinary observed in the atmosphere, but again it is difficult to explain the correlation.

A different explanation: coherent particle scatter.

A completely different explanation could be that the correlation in the reflectivities is caused by coherent particle scatter and variations in the concentration in different resolu-

Fig.1. Scatter plot of X band versus UHF band reflectivities for the smoke measurement described in [1].

tion cells. For low concentrations the first term of (8) will dominate at both wavelengths and the difference in reflectivities will be 0 dB. For high concentrations the second term will dominate at both wavelengths, resulting in a 37 dB difference. In between, there is a range of concentrations for



which the first term will dominate at X-band, while the second term will dominate at UHF-band. Variations in concentration in this range will lead to a slope of 2 in a scatterplot of reflectivities. That is close to the slope that has been observed. Variations in the diameters of the particles would give a slope of 1. If our explanation is valid, than the variations in the diameter cannot be very large.

There is large uncertainty in the values of β and L_0. A value of 10 m has been used by VanZandt, *et al.* [8]. If β is taken to be unity, then it can be calculated with (8) that concentrations ranging from 1e5 m^{-3} to 1e7 m^{-3} and a diameter of about 100 μm are needed to explain Fig. 1. For lack of better knowledge, $|K|^2$ was taken to be 0.93.

Developing cumulus clouds

Another case where there was a puzzling correlation between the radar returns of two radars is described in the article of Knight and Miller [2]. They observed developing cumulus clouds with a 3 cm X-band radar and a 10 cm S-band radar simultaneously. If the returns would both be dominated by the same scattering mechanism, the returns would be correlated and have a 0 dBZ difference if incoherent scattering is the dominating mechanism, or 19 dBZ if coherent scattering is most important at both wavelengths. However, often there was a correlation between the two returns, but the difference in reflectivity was about 10 dBZ. No satisfactory explanation was found. A correlation between coherent scattering from the air and incoherent scattering from droplets is not to be expected. We strongly suspect that coherent scattering *from the droplets* dominates at S-band and incoherent scattering at X-band. The fluctuations in liquid water content may be caused by mixing of droplet free air from outside the cloud with the cloudy air. The correlations are caused by variations in diameter, which are due to the growth of ascending droplets.

Gossard and Strauch [6] have calculated that coherent scattering from water vapor fluctuations is about 30 times as strong as coherent scattering from fluctuations in liquid water content, if the fluctuations are equally large. If the observed correlation in the X-band and S-band returns is due to fluctuations in liquid water content, it must be explained why these fluctuations are much larger than those of water vapor content. A combination of several mechanisms can cause that. Mixing with air from outside the cloud can produce large fluctuations in liquid water content, because the outside air does not contain droplets. If the air outside the cloud is also very humid, then the fluctuations in liquid water content may be larger than those in water vapor content. In addition, condensation and evaporation will always reduce the fluctuations in water vapor. Furthermore, variations in liquid water on small scales may exist for much longer times than those of water vapor, because diffusion of liquid water is almost absent. However, fluctuations in liquid water can be smoothed out by differences in fall velocity of and inertia of the droplets – the broader the droplet size distribution, the more important these are.

CONCLUDING REMARKS

An explanation was proposed of recently published dual frequency measurements, in terms of coherent scatter from particles. Coherent particles scatter may be present in other cases as well. There are indications that coherent scatter can be important in the melting layer [9].

The values of the parameters and constant in (8) are highly uncertain. We are also not sure whether the turbulence theory can be applied if fluctuations are so large that they are comparable in size to the mean. It may be necessary to perform experiments under controlled conditions to see whether coherent particle scatter is really significant.

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Erratum

Two constants are wrong. Equation (4) should be:

$$C_n^2 \approx 5.5\, L_0^{-2/3}\, var\, n$$

And equation (8) should be:

$$Z \times 10^{-18} = ND^6 + 4.2 \times 10^{-3} \boldsymbol{b}^2 L_0^{-2/3} N^2 D^6\, \boldsymbol{l}^{11/3}$$