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COHERENT PARTICLE SCATTER IN CLOUDS: REFLECTION CALCULATIONS BASED ON IN-SITU MEASUREMENTS

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Well-known radar scattering mechanisms are Rayleigh scattering (*incoherent particle scattering*), and clear-air scattering (*coherent air scattering*). This article discusses the importance in clouds of these two mechanisms compared to a third scattering mechanism: *coherent particle scattering*, which is caused by spatial variations in the liquid water content of the clouds. We will argue that for radars with a wavelength larger than a centimetre, coherent particle scatter can dominate Rayleigh scattering from individual droplets, both for cumulus and stratiform clouds. Coherent air scatter in clouds is probably less important than previously thought.

Incoherent particle scatter is due to particles that are randomly distributed within the radar volume. It is most important in atmospheric radars with a small wavelength. Coherent air scatter is caused by variations in the refractive index of atmospheric gases. It is normally dominated by spatial variations in humidity (Gossard and Strauch, 1980) and occurs mainly in radar measurements using a long wavelength (cm-waves or longer).

Erkelens et al. (2000) recently showed that coherent particle scatter, which is caused by variations in the liquid water content (LWC) of clouds, may be important in clouds. Like coherent air scatter this mechanism is mainly important in atmospheric radars with a long wavelength. Erkelens et al. showed that coherent particle scatter can explain the results of dual-wavelength radar measurements of cumulus clouds performed by Knight and Miller (1998) and those of a smoke plume done by Rogers and Brown (1997). In this article we explore the importance of this scattering mechanism for other radar systems and cloud types. Coherent particle scatter is compared in strength to the other two scattering mechanisms. As quantitative data is sparse, the discussion must be labelled as tentative, but for the sake of readability this is not stressed continuously.

2. THEORY OF COHERENT PARTICLE SCATTER

For a cloud with a mono-disperse drop size distribution with diameter (D) in which the particle mass variations are transported from large scales to small scales by isotropic homogeneous turbulence, the radar reflectivity factor (Z) is given by (Erkelens et al., 2000):

$$Z = ND^6 + 4.2 \times 10^{-3}\, b^2 L_0^{-2/3} N^2 D^6\, l^{11/3} \qquad (1)$$

with N the ensemble average particle number density, L_0 the outer scale length of the inertial subrange of isotropic homogeneous turbulence, and l the radar wavelength. The first term is the incoherent backscatter and the second term the coherent backscatter. The standard deviation of the spatial Liquid Water Content (LWC) variations is assumed to be a fraction (b) of the total LWC. The derivation is only briefly described in this article – as Erkelens et al. already did this in depth – and is new in the sense that the slope of the LWC variance spectrum is allowed to have various values.

In Van de Hulst (1981) the refractive index (n) of air with many small spheres is formulated as:

$$n = 1 + \frac{p}{4} K D^3 N \qquad (2)$$

with K = $(e_r - 1)(e_r + 2)^{-1}$, a constant that is determined by the relative permittivity of the particles (e_r); the absorption is neglected.

Assuming that the standard deviations of the variations are a fraction of the mean LWC, i.e. $s_{LWC} = b \cdot LWC$, or $\mathrm{var}(ND^3) = b^2 (ND^3)^2$, one can write:

$$\mathrm{var}\, n = \frac{p^2}{16} |K|^2\, \mathrm{var}(ND^3) = \frac{p^2}{16} |K|^2\, b^2 N^2 D^6 \qquad (3)$$

The radar backscatter is determined by the variance of the three-dimensional variance density spectrum ($f_n(\boldsymbol{k})$) in a small spectral band around half the radar wavelength in the radars direction ($\hat{\boldsymbol{k}}$). Following Ottersten (1969) we assume that the spatial variance spectrum integrated over the entire wave number (k) space is equal to the total spatial variance of the refractive index (var n):

$$\int f(\boldsymbol{k})\, d\boldsymbol{k} = \mathrm{var}\, n \qquad (4)$$

To compute this three-dimensional power density integral, it is assumed that the energy spectrum of the LWC variations follows a power law.

Slopes different from –5/3 (the value for homogeneous isotropic turbulence in the inertial subrange) have been measured in clouds. That is why an equation for coherent particle scatter for slopes between –1 and –3 is derived here. According to Tatarski (1961), the three-dimensional spectral density (f(k)) for isotropic turbulence is in this case given by:

$$f(k) = \frac{\Gamma(p+1)}{4p^2} \sin\left(\frac{p(p-1)}{2}\right) C_n^2 k^{-(p+2)} \qquad (5)$$

with Γ(..) the gamma function, $-p$ the slope of the (LWC) variance spectrum. The structure constant of the refractive index (C_n^2) is a measure for the total amount of refractive index variations per unit volume.

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L_0 (outer scale) and l_0 (inner scale) are, respectively, the largest scale and the smallest scale of the inertial subrange. Equation (4) can be computed by using Eq. (5) and by assuming that there are no variations at scales larger than L_0 or smaller than l_0. C_n^2 is then given by:

$$C_n^2 = \frac{p-1}{2\Gamma(p+1)} \frac{(2p)^p}{\sin(p(p-1)/2)} \left(L_0^{p-1} - l_0^{p-1}\right)^{-1} \operatorname{var} n \quad (6)$$

The relation between C_n^2 and the radar reflectivity can be found by combining the expression,

$$\eta = \frac{\pi^2}{2} k^4 \phi(k) \quad (7)$$

from Ottersten (1969) with Eq. (5):

$$\eta = \frac{\Gamma(p+1)}{8} \sin(\pi(p-1)/2) k^{-p+2} C_n^2 \quad (8)$$

Combining this with Eq. (6) and substituting $k=4\pi\lambda^{-1}$ yields the radar reflectivity as a function of var n:

$$\eta = \frac{(p-1)\pi^2}{2^p} \left(L_0^{p-1} - l_0^{p-1}\right)^{-1} \lambda^{p-2} \operatorname{var} n \quad (9)$$

The radar reflectivity factor is defined by:

$$Z = \frac{\lambda^4}{\pi^5 |K|^2} \eta \quad (10)$$

Finally, by combining (3), (9), and (10), the radar reflectivity factor for coherent particle scatter for a variance spectrum with $1<p<3$ is found:

$$Z = \frac{p-1}{16\pi\ 2^p} (L_0^{p-1} - l_0^{p-1})^{-1} \lambda^{p+2} N^2 b^2 D^6 \quad (11)$$

which will reduce to the second term on the right-hand side of Eq. (1) for p=5/3 and $l_0 \ll L_0$. For most slopes the term with the inner scale (l_0) can be ignored, as the outer scale (L_0) is normally much bigger. However for p close to 1 the inner scale will become important.

3. SPATIAL VARIATIONS IN CLOUDS

A literature review of measured spatial variations and its sources and sinks is given in Venema et al. (1999). Davis et al. (1999) have measured LWC variations in stratocumulus clouds of 0.290 ± 0.167 g/m^{-3}, so the relative standard deviation is 58 %. Davis et al. (1996) found an average relative standard deviation of LWC of about 19 % in stratocumulus, but it varied highly per measurement; the lowest value found was 5 % and the highest 25 %. Politovich and Cooper (1988) have measured *humidity* variations in cumulus clouds; in the entrained regions the standard deviation was below 0.4 % and in the core of the cloud around 0.1 %.

For fully developed homogeneous isotropic turbulence the slope is –5/3 in the inertial subrange if the sources of the variations act on scales larger than the outer scale (L_0) and the sinks act on scales smaller than the inner scale (l_0). For a passive *conservative* additive the slope of the variance spectrum of the additive is the same as the slope of the turbulent energy spectrum (Tatarski, 1961). However, both the amount of water vapour and liquid water are not conserved in clouds. One can expect the slope of the humidity or LWC variance spectrum to become more flat (steep) if there is an additional source (sink) of variations within the inertial subrange. Slopes that have been measured in situ or by radar are summarised in table 1. Note, that the slopes measured by dual wavelength radar may be incorrect as both the wavelengths may not lie in the inertial subrange

4. COHERENT PARTICLE AND COHERENT AIR SCATTER

Gossard and Strauch (1983) calculated that if the spatial variance of the water vapour content is equal to the spatial variance of liquid water content, the radar reflectivity due to the humidity variations should be about 28 times larger. The quantitative relations are:

$$10^{12} \cdot \operatorname{var} n_L = 2.09 \cdot \operatorname{var} L$$
$$10^{12} \cdot \operatorname{var} n_V = 58.5 \cdot \operatorname{var} V \quad (12)$$

with, respectively, var n_L and var n_V the variance of the refractive index due to Liquid water variations (var L) and water Vapour variations (var V). In other words, if the coherent particle scatter is to dominate the coherent air scatter, the spatial *standard deviation* of the LWC variations should be at least 5.3 times larger than the spatial standard deviation of the humidity variation.

The relative standard deviations in the spatial humidity variations measured by Politovich and Cooper (1988) in cumulus clouds are: 0.4 % and 0.1 %, see above. One can calculate that the magnitude of the relative liquid water variations in cumulus clouds can be easily more than 5.3 times, except near the cloud base [Venema et al., 1999]. In stratiform clouds the difference is much smaller and measurements should be made to attain a reliable answer.

In the above calculations the slopes of the LWC and humidity variations are assumed to be equal. If this is not the case, the results can be drastically different. Assuming that $l_0 \ll L_0$, and using Eq. (9) and Eq. (12)one can derive the following:

Cloud type	Slope	Additive	Measurement method	Source
Stratocumulus	–0.94 ± 0.10	LWC	In situ, at scales below 5 m	Davis et al. (1999)
Stratocumulus	–1.6 ± 0.1	LWC	In situ, at scales above 5 m	Davis et al. (1999)
Stratocumulus	–1.36±0.06	LWC	In situ	Davis et al. (1996)
Cumulus	–2.2	Humidity	Dual wavelength radar	Knight & Miller (1998)
Winter clouds	–0.9	LWC/hum.	Forward scattering radar	Gossard & Strauch (1981)
Low level clouds	–1.1 to –1.3	LWC/hum.	Dual wavelength radar	Gage et al. (1999)

Table 1. Slopes of the variance spectra in clouds.

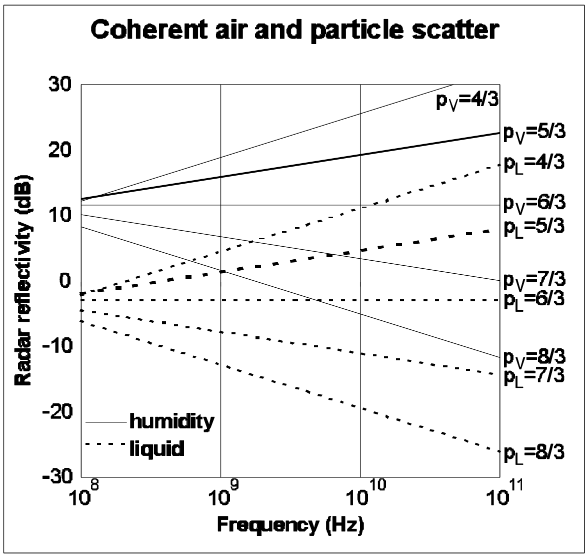

Figure 1. Calculation of the strength of coherent air and coherent particle scatter for different slopes of the humidity and LWC variance spectrum, using Eq. (11) and (13). The dashed line is the radar reflectivity from LWC variations and the drawn line from the humidity variations. The variances in humidity and LWC are equal, and L_0 is 10 m.

$$\frac{h_L}{h_V} = \frac{p_L - 1}{p_V - 1} 2^{-p_L + p_V} L_0^{-p_L + p_V} l^{p_L - p_V} \frac{\operatorname{var} L}{28 \operatorname{var} V} \tag{13}$$

with η_L and η_V the radar reflectivity due to LWC variations and humidity variations, respectively, and $-p_L$ and $-p_V$ the slopes of respectively the LWC and humidity variance spectra.

In Fig. 1 η_L and η_V are compared for various slopes and wavelengths for identical variance of humidity and LWC and L_0 = 10 m. For a larger value of L_0 the differences will be larger. In Fig. 1 one can see that for equal slopes the coherent air scatter is 28 times (15 dB) more than the coherent particle scatter, as expressed by Eq. (12). When the slope of the humidity variations is steeper than that of the LWC variations, the lines can cross. For example, the radar reflectivity due to humidity variations (for a slope $-p_V$ of –2.2) and the radar reflectivity due to the LWC variations (for a slope $-p_L$ of –1.36) will be almost equal for an S-band radar (λ=0.1 m), in-stead of 28 times smaller.

5. COHERENT AND INCOHERENT PARTICLE SCATTER

To investigate how important incoherent scatter is compared to coherent particle scatter for various scenarios we illustrate the influence of the variables in Eq. (1) and (11) with a few graphs. Fig. 2a shows the relative strength of coherent particle scatter compared to incoherent particle scatter for different values of the relative spatial standard deviation of LWC (*b*).The high values may occur at cloud boundaries and the low ones in a cloud that is not very turbulent.

In Fig. 2b the relative strength of coherent particle scatter compared to incoherent particle scatter is investigated in relation to the number density (*N*). A number density of 1000 cm^{-3} is a high value for cumulus, 200 to 500 cm^{-3} is representative for stratus clouds, and the value of 10 cm^{-3} corresponds to some ice clouds. The other parameters are shown in the figure.

The value for the outer scale is uncertain; fortunately it is a less importance (Fig. 2c). VanZandt et al. (1978) used 10 m. L_0 is largest in turbulent regions with low hydrostatic stability (Gage, 1999). We included this graph with L_0 as independent variable to show that the spread is not very large compared to the other variables.

The influence of the slope (*p*) is plotted in Fig. 2d, using Eq. (11). The inner scale is assumed zero. The difference in reflectivity between the plotted slopes will become larger when a larger outer scale is chosen.

Concluding, the strength of coherent scatter compared to incoherent scatter is largely determined by the radar wavelength. For wind profilers coherent particle scatter will often dominate incoherent particle scatter. For typical number densities in ice clouds only a wind profiler may receive significant coherent scatter. For typical number densities and relative LWC variations found in stratocumulus clouds, coherent particle scatter (either from humidity or LWC variations) can dominate incoherent scatter for an S-band radar. For an X-band radar, coherent particle scatter can dominate only for the highest values of β and number densities. For mm-wave radars coherent particle scatter is not likely, although given the uncertainty in the variables (especially *b* and *p*) it cannot be ruled out. Note that the inner scale may be larger than half the wavelength in some situations. When the slope is flatter than –5/3, which occurs, as measurements have shown, the coherent particle scattering is several dBs stronger at typical radar wavelengths (see Fig. 2d).

6. CONCLUSIONS & RECOMMENDATIONS

This paper explored the possibility of significant coherent particle scatter in clouds. Measurements in literature have shown significant spatial variations in liquid water content and humidity in stratus, stratocumulus and cumulus clouds.

There is evidence from dual-wavelength measurements that the slope of the humidity spectrum can be steeper than the standard value of –5/3, while evidence from in situ measurements shows that the slope of the LWC spectrum can be flatter. This can have a significant impact on radar reflectivity.

Calculations show that coherent particle scatter can dominate coherent air scatter and incoherent particle scatter in the top part of cumulus clouds. In stratocumulus clouds coherent scatter can dominate, to determine whether the source of the coherent scatter is variations in LWC or humidity, simultaneous in-situ measurements of these parameters are needed.

To test the theory of coherent particle scatter, simultaneous measurements of LWC and humidity variations should be compared to measurements of coherent radar scatter.

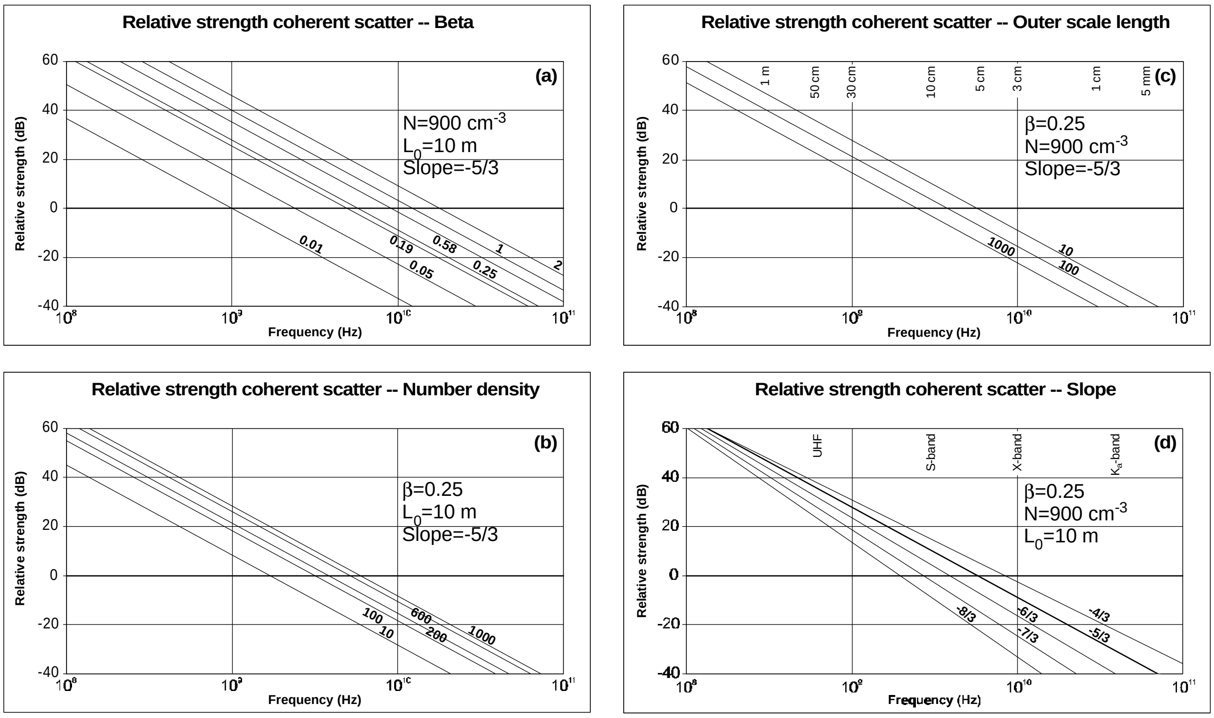

Figure 2. Plots of the relative strength of coherent particle scatter compared to incoherent particle scatter as a function of radar frequency. On the 0-dB level both mechanisms are equally strong.

To develop the theory further, small-scale in situ measurements should be made of humidity, LWC and temperature spectra, both of the (co-)variance and the slopes. Measurements of the LWC and humidity spectra should be made for a range of different cloud types, to determine which type of coherent scatter is important for which type of cloud. A theoretical expression for coherent particle scatter for LWC spectra with a slope flatter than –1 should be developed, especially since a slope of –0.9 has been observed.

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